Effects of Nanoparticle Clustering on the Heat Transport in Nanofluids Through Fractal Theories

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Considering that the motion of microphysical object takes place on continuous but non-differentiable curves, i.e. on fractals, effects of nanoparticle clustering on the heat transfer in nanofluids using the scale relativity theory in the topological dimension $D_T = 3$ *are analyzed. In the one-dimensional differentiable case, the clustering morphogenesis process is achieved by cnoidal oscillation modes of the speed field and a relation between the radius and growth speed of the cluster is obtained. In the non-differentiable case, the fractal kink spontaneously breaks the vacuum symmetry by tunneling and generates coherent structures. Since all the properties of the speed field are transferred to the thermal one and the fractal potential (fractal soliton) acts as an energy accumulator, for a certain condition of an external load (e.g. for a certain value of thermal gradient) the fractal soliton breaks down (blows up) and releases energy. As result, the thermal conductibility in nanofluids unexpectedly increases.*

Keywords: nanofluid, heat transfer, fractal, cluster

Nanofluid is a new kind of heat transfer medium containing nanoparticles which are uniformly and stable distributed in a base fluid. Experiments on nanofluids have demonstrated that the thermal conductivity increases with decreasing grain size [1-4]. Keblinski *et al* [4] have examined four possible mechanisms for the anomalous enhancement observed in nanofluids: Brownian motion of the nanoparticles [5-7], molecular-level layering of the liquid at the liquid-nanoparticles interface [8], the effects of nanoparticle clustering [9], and ballistic phonons transport [4].

Recently, the increasing of the heat transfer in nanofluids was related to the fractal effects [10, 11]. Moreover, Wang *et al.* [12] reported that the modified fractal model agreed well with the experimental data obtained for the SiO_2/ ethanol nanofluid. In such conjecture, the fractal theories (particularly the scale relativity theory (SRT) [13,14] is a new approach to understand quantum mechanics, and furthermore physical domains involving scale laws, such as the nanosystems [15,16, 17]. It is based on a generalization of Einstein's principle of relativity to scale transformations. Namely, one redefines space-time resolutions as characterizing the state of scale of reference systems, in the same way as velocity characterizes their state of motion. Then one requires that the laws of physics apply whatever the state of the reference system, of motion (principle of motion-relativity) and of scale (principle of SRT). The principle of SRT is mathematically achieved by the principle of scale-covariance, requiring that the equations of physics keep their simplest form under transformations of resolution. For example, considering that the motion of micro-particles take place on continuous but non-differentiable curves, *i.e.* on fractals [13, 14], it was demonstrated that, in the topological dimension [18] D_T=2, the geodesics of the fractal space-time are given by a Schrödinger's type equation.

In the present paper, using the SRT, we analyzed the effects of nanoparticle clustering on the heat transfer in nanofluids.

Theoretical part

Mathematical model

A non-differentiable continuum is necessarily fractal and the trajectories in such a space (or space-time) own (at least) the following three properties:

i) the test particle can follow an infinity of potential trajectories: this leads one to use a fluid-like description;

ii) the geometry of each trajectory is fractal (of dimension 3 – for other details on the fractal dimension see [15, 16]. Each elementary displacement is then described in terms of the sum, $d\mathbf{X} = d\mathbf{x} + d\zeta$ of a mean classical displacement $d\mathbf{x} = \mathbf{v}dt$ and of a fractal fluctuation $d\zeta$, whose behaviour satisfies the principle of SRT (in its simplest Galilean version). It is such that $\langle d\zeta \rangle = 0$ and $\langle d\zeta^3 \rangle = (6D^2 / c)\, dt$ where D defines the fractal/non-fractal transition, *i.e.* the transition from the explicit scale dependence to scale independence and c is the light speed in vacuum. The existence of this fluctuation implies introducing new third order terms in the differential equation of motion;

iii) time reversibility is broken at the infinitesimal level: this can be described in terms of a two-valuesness of the velocity vector for which we use a complex representation, $\mathbf{V} = (\mathbf{v}_+ + \mathbf{v}_-) / 2 - i(\mathbf{v}_+ - \mathbf{v}_-) / 2$. We denoted by $\mathbf{v}_+$ the "forward" speed and by v_- the "backward" speed.

These three effects can be combined to construct a complex time-derivative operator (Appendix A)

$$\frac{\delta}{dt} = \frac{\partial}{\partial t} + V \cdot \nabla + \frac{D^2}{c} \nabla^3 \qquad (1)$$

Now, the first Newton's principle in its covariant form, $\delta\mathbf{V} / dt = 0$, becomes

$$\frac{\delta V}{dt} = \frac{\partial V}{\partial t} + V \cdot \nabla V + \frac{D^2}{c} \nabla^3 V = 0 \qquad (2)$$

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i.e. **a Korteweg – de Vries (KdV) type equation in a fractal space-time**. This means that, both of the differential scale and the fractal one, the complex acceleration field, $\delta \mathbf{V}$ / dt, depends on the local time dependence, $\partial_t \mathbf{V}$, on the non-linearity (convective) term, $V \cdot \nabla V$, and on the dispersive one, $\nabla^3 V$. Moreover, the behaviour of a "non-differentiable fluid" is viscoelastic or hysteretic type. The theory regarding the viscoelastic structural behaviour applied at macroscopic level was used to the creep [19] and relaxation [20] modeling for some polymeric type materials.

Such a result is in agreement with the opinion given in [9, 15, 16]: the non-differentiable fluid can be described by Kelvin-Voight model or Maxwell rheological model with the aid of complex quantities e.g. the complex speed field, the complex acceleration field etc.

From (2) and by the operational relation $V \cdot \nabla V = \nabla(V2 / 2) - V \times (\nabla \times V)$ we obtain the equation:

$$\frac{\delta V}{dt} = \frac{\partial V}{\partial t} + \nabla\left(\frac{V^2}{2}\right) - V \times (\nabla \times V) + \frac{D^2}{c}\nabla^3 V = 0 \tag{3}$$

If the motions of the "non-differentiable fluid" are irrotational, *i.e.* $\Omega = \nabla \times V = 0$ we can choose V of the form:

$$V = \nabla \phi \tag{4}$$

with ϕ a complex speed potential. Then, equation (3) becomes:

$$\frac{\delta V}{dt} = \frac{\partial V}{\partial t} + \nabla\left(\frac{V^2}{2}\right) + \frac{D^2}{c}\nabla^3 V = 0 \tag{5}$$

and more, by substituting equation (4) in equation (5), we have

$$\nabla\left(\frac{\partial \phi}{\partial t} + \frac{1}{2}(\nabla\phi)^2 + \frac{D^2}{c}\nabla^3\phi\right) = 0 \tag{6}$$

This yields:

$$\frac{\partial \phi}{\partial t} + \frac{1}{2}(\nabla\phi)^2 + \frac{D^2}{c}\nabla^3\phi = F(t) \tag{7}$$

with F(t) a function of time only. We realize that (5) have been reduced to a single scalar relation (7), i.e. a Bernouilli-type equation.

If ϕ simultaneously becomes complex speed potential and wave-function, i.e.$\phi = -2iD\ln\psi$, with D the Nottales' coefficient [13,14], equation (7), up to an arbitrary phase factor which may be set to zero by a suitable choice of the phase of ψ i.e.F(t) = 0, implies the non-linear Schrödinger type equation:

$$D^2\Delta\psi + iD\partial_t\psi + \left(-2D^2\Delta \ln\psi + \frac{iD^3}{c}\nabla^3 \ln\psi\right)\psi = 0 \tag{8}$$

Results and discussion

Effects of nanoparticle clustering at differentiable scale

Let us consider the relation (4) in the form:

$$V = v + iu \tag{9}$$

According to our previous observations, $\boldsymbol{v}$ will correspond to the classical speed given by the differential part of $\boldsymbol{V}$, and $\boldsymbol{u}$ will correspond to the fractal speed given by the non-differential part of $\boldsymbol{V}$. By replacing (9) in equation (5) and separating the real part from the imaginary one, we obtain the following system:

$$\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v}\nabla\boldsymbol{v} - \boldsymbol{u}\nabla\boldsymbol{u} + \frac{D^2}{c}\nabla^3\boldsymbol{v} = 0$$
$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{v}\nabla\boldsymbol{u} + \boldsymbol{u}\nabla\boldsymbol{v} + \frac{D^2}{c}\nabla^3\boldsymbol{u} = 0 \tag{10a,b}$$

In the differentiable case, i.e. u = 0, the system (10a, b) becomes:

$$\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v}\nabla\boldsymbol{v} + \frac{D^2}{c}\nabla^3\boldsymbol{v} = 0 \tag{11}$$

Considering that the heat transfer process in nanofluids is one-dimensional [8-12], equation (11) takes the standard form of the KdV equation [18]:

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + \frac{D^2}{c}\frac{\partial^3 v}{\partial x^3} = 0 \tag{12}$$

Using the dimensionless parameters [11, 12], $\phi = (v/v_0)$, $\tau = \omega_0 t$, $\xi = k_0 x$ and the normalizing conditions [11],$k_0 v_0 = D^2 k_0^3 / c = 6\omega_0$ equation (12) becomes:

$$\partial_\tau \phi + 6\phi\partial_\xi\phi + \partial_{\xi\xi\xi}\phi = 0 \tag{13}$$

Through the substitutions,$w(\theta) = \phi(\xi,\tau)$, $\theta = \xi - u\tau$ where (ω_0, k_0, v_0) are parameters characterizing the critical growth speed field of the cluster (for other details see [11, 12]), the equation (13), by double integration, becomes

$$\frac{1}{2}w'^2 = F(w) = -(w^3 - \frac{u}{2}w^2 - gw - h) \tag{14}$$

with g, h two integration constants. If $F(w)$ has real roots, they are of the form

$$e_1 = 2a\frac{E(s)}{K(s)},\ e_2 = 2a\left[\frac{E(s)}{K(s)} - 1\right], e_3 = 2a\left[\frac{E(s)}{K(s)} - \frac{1}{s^2}\right] \tag{15a-c}$$

with

$$a = \frac{e_1 - e_2}{2},\ s^2 = \frac{e_1 - e_2}{e_1 - e_3},\ K(s) = \int_0^{\pi/2}(1 - s^2\sin^2\varphi)^{-1/2}d\varphi,$$
$$E(s) = \int_0^{\pi/2}(1 - s^2\sin^2\varphi)^{1/2}d\varphi \tag{16a-d}$$

and $K(s)$, $E(s)$ the complete elliptic integrals [21]. Then, the solution of equation (13) has the expression

$$\phi(\xi,\tau) = 2a\left(\frac{E(s)}{K(s)} - 1\right) +$$
$$+ 2acn^2\left\{\sqrt{a}s^{-1}\left[\xi - 2a\left(\frac{3E(s)}{K(s)} - \frac{1+s^2}{s^2}\right)\tau + \xi_0\right]; s\right\} \tag{17}$$

where cn is the Jacobi's elliptic function of s modulus [21] and ζ_0 constant of integration. As a result, the clustering morphogenesis process is achieved by one-dimensional cnoidal oscillation modes of the speed field. This process is characterized through the normalized wave

$$\lambda = \frac{2sK(s)}{\sqrt{a}} \tag{18}$$

- see figure 2, and the normalized phase speed

$$u = 4a\left[3\frac{E(s)}{K(s)} - \frac{1+s^2}{s^2}\right] \tag{19}$$

- see figure 3. Then:

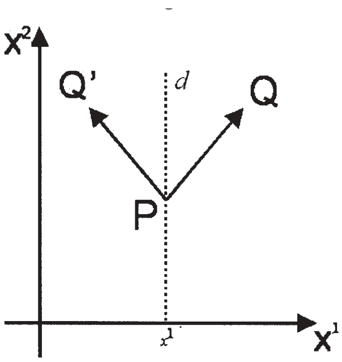

Fig. 1 The continuous curves which are not fractals but have points where they are not differentiable

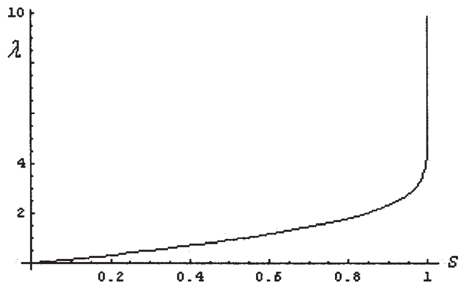

Fig. 2. The dependence of the normalized wave length λ with s

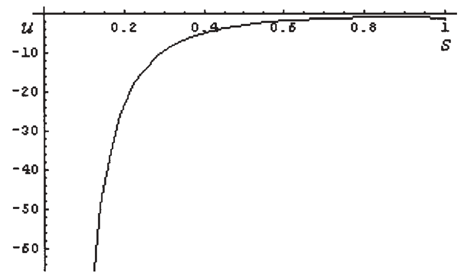

Fig. 3. The dependence of the normalized phase speed u with s.

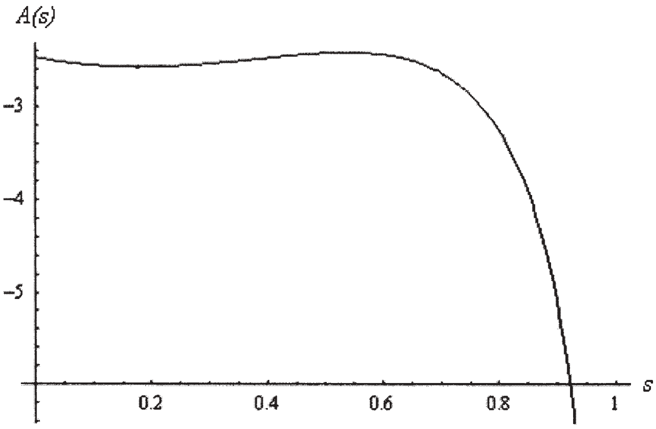

Fig. 4. The dependence $A = A(s)$

i) the parameter λ from equation (18) corresponds through $\lambda \equiv R$ to the normalized radius of the cluster, and *u* from equation (19) to the normalized growth speed of the cluster. Moreover, by eliminating the parameter *a* from relations (18) and (19), one obtains

$$uR^2 = A(s),\ A(s) = 16\left[3s^2E(s)K(s) - (1+s^2)K^2(s)\right] \quad (20)$$

where the quantity A(s) is numerically evaluated.

For $s = 0 - 0.7$, $A(s) \approx$ const. (fig. 4 and equation 20) takes the form,

$$uR^2 = const. \quad (21)$$

ii) through the D coefficient, the parameter *s* becomes a measure of the heat transfer in nanofluids. Thus, for an increased of the heat transfer, i.e. $s \to 0$, the normalized growth speed (*u*) of the cluster is high and the normalized radius (*R*) of the cluster is small (fig. 2 and 3). On the contrary, for a decreased of the heat transfer in nanofluids, i.e. $s \to 1$, *u* is small and *R* is high (fig. 2 and 3);

iii) the one-dimensional cnoidal speed oscillation modes contain as subsequences, for s = 0 the one-dimensional speed harmonic waves, for $s \to 0$ the one-dimensional speed waves packet and for $s \to 1$ the one dimensional speed solitons packet.

These subsequences describe the clustering morphogenesis process.

For $s \to 1$, the solution (17), with the substitutions $\phi_0 = e_3$ and $k^2 = (e_1 - e_3)/2$, becomes the one-dimensional speed soliton

$$\phi(\xi,\tau) = \phi_0 + 2k^2 \sec h^2\left[k\left(\xi - \left(4k^2 + 3\phi_0\right)\tau + \xi_0\right)\right] \quad (22)$$

of amplitude $2k^2$, width k^{-1} and phase velocity $u = 4k^2 + \phi 3_o$ This subsequence describes the cluster as a quasi-autonomous structure.

Effects of nanoparticle clustering at non-differentiable scale

In the non-differentiable case the equation (14), with the substitutionsw = $(u/4)f^2$ and $i\eta = (u/4)^{1/2}\theta$ and with the restriction h = 0, becomes a Ginzburg-Landau type equation [18]:

$$\partial_{\eta\eta} f = f^3 - f \quad (23)$$

It results:

i) the η coordinate has dynamic significations and *f* variable, probabilistic ones (for details see also [13, 14]). The space-time becomes a fractal one (for details see [13, 14]) the fluid acquires fractal properties (fractal fluid by short);

ii) according to [22] we can build a field theory with spontaneous symmetry breaking. The fractal kink,

$$f_k(\eta) = f(\eta - \eta_0) = tanh\left[\frac{1}{\sqrt{2}}(\eta - \eta_0)\right] \quad (24)$$

spontaneously breaks the vacuum symmetry by tunneling and generates coherence structures. This mechanism is similar with the one of superconductivity [23];

iii) through an analogy with the Bohm's potential [24], the fractal potential takes a very simple expression which is directly proportional to the states density of the fractal fluid, *i.e.*

$$Q = -\frac{2mD^2}{f}\frac{d^2 f}{d\eta^2} = 2mD^2\left(1 - f^2\right) \quad (25)$$

When the states density, f^2, becomes zero, the fractal potential takes a finite value, $Q = 2mD^2$. The fractal fluid is normal and there are no coherent structures in it. When f^2 becomes 1, the fractal potential turns to zero, the entire quantity of energy of the fractal fluid is transferred toward its coherent structures. Then the fractal fluid becomes coherent through self-structuring. Therefore, one can assume that the energy from the fractal fluid can be stocked by transforming all the environment's entities into coherence structures and then 'freezing' them. The fractal fluid act as an energy accumulator through the fractal potential;

iv) substituting (24) in (25) the fractal potential becomes the fractal soliton (a soliton in a fractal space-time)

Conclusions

The main conclusions of the present paper are the following:

i) through the scale relativity theory in the topological dimension $D_T = 3$, in the differentiable case the clustering morphogenesis process is achieved by one-dimensional cnoidal oscillation modes of the speed field;

ii) for different degrees of the heat transfer in nanofluids, the one-dimensional cnoidal speed oscillation modes contain the one-dimensional speed harmonic waves, the one-dimensional speed waves packet, the one dimensional speed solitons packet and the one dimensional speed soliton. The first three subsequences describe the dynamics of the cluster, while the last one describes the cluster as a quasi-autonomous structure;

iii) a relation between the normalized radius and the normalized growth speed of the cluster is obtained;

iv) in the non-differentiable case we can build a field theory with spontaneous symmetry breaking. The fractal kink spontaneously breaks the vacuum symmetry by tunneling and generates coherent structures. Moreover, the fractal fluid acts as an energy accumulator through the fractal potential (fractal soliton);

v) usually, the speed field is proportional to a square root of a normalized temperature (for details see [9-11]). Then, all the properties of the speed field are transferred to the thermal one. In certain conditions of an external load (e.g. for a certain value of thermal gradient) the soliton breaks down (blows up) and releases energy.

As result, the thermal conductibility in nanofluides unexpectedly increases.

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